Int J Theor Phys (2009) 48: 3331–3338
DOI 10.1007/s10773-009-0135-6

Schemes for Splitting Quantum Information via Tripartite Entangled States

Qun-Yong Zhang · You-Bang Zhan · Ling-Ling Zhang · Peng-Cheng Ma

Received: 29 June 2009 / Accepted: 26 August 2009 / Published online: 12 September 2009

Abstract In this paper we propose two schemes for quantum information splitting via tripartite entangled states. Explicit protocols for the quantum information splitting of a single qubit state and an arbitrary two-qubit entangled state are illustrated. We also consider the security against certain eavesdropping attacks. Moreover, a generalization of the scheme to multi-particle case is also outlined.

Keywords Quantum information splitting · Teleportation · Tripartite entangled state

1 Introduction

Quantum communication plays a significant role in the ongoing field of information theory. It is well known that novel phenomena including quantum teleportation [1], quantum dense coding [2], quantum key distribution [3], quantum secret sharing [4], and so on. Teleportation is a technique for transfer of information between parties, using a distributed entangled state and a classical communication channel. In classical secret sharing [5], a secret message can be distributed among N users in such a way that, only by combining their pieces of information can the N users recover the secret message. In quantum secret sharing (QSS) the owner who possesses and wishes to transmit the secret information splits it among various parties such that the original information can only be reconstructed by a specific subset of the parties. The quantum version of secret sharing cannot only provide absolute security, but also likely play a key role in protecting secret quantum information.

Hillery et al. [4] have described a procedure for realizing quantum secret sharing by using multiparticle maximally entangled states, e.g., three-particle and four-particle Greenberger-

Q.-Y. Zhang · L.-L. Zhang
School of Physical Science and Technology, Suzhou University, Suzhou 215006, China

Q.-Y. Zhang · Y.-B. Zhan (✉) · L.-L. Zhang · P.-C. Ma
School of Physics and Electronic Electrical Engineering, Huaiyin Normal University, Huaian 223300, China
e-mail: ybzhan@hytc.edu.cn

Horne-Zeilinger (GHZ) states [6]. Karlsson et al. [7] and Cleve et al. [8] have proposed different schemes for QSS which require the particle carrying the quantum information be first entangled with the other particles to share the information. Since then, many QSS schemes were presented [9–16]. Recently, Lance et al. [17] proposed a scheme for QSS of quantum information, called as the quantum state sharing (QSTS), where the secret is an unknown quantum state, to differentiate from the QSS of classical information. QSTS is also usually named as quantum information splitting (QIS), and various kinds of QIS (or QSTS) schemes have been presented [18–28].

Quantum entanglement is one of the most striking feature of quantum mechanics. Also multipartite entanglement is a very important physical resource in quantum information processing. So far, tripartite entanglement has been well studied theoretically and experimentally [29–40]. In this paper, we present a scheme for splitting quantum information using a three-qubit entangled state as quantum channel. Explicit protocols for quantum information splitting (QIS) of single- and two-qubit state will be illustrated. Finally, the scheme is generalized to tripartite QIS of an arbitrary N-particle state via N tripartite entangled state.

2 QIS of a Single-Particle State via a Tripartite Entangled State

We assume that Alice possesses a qubit, which is in an unknown state

$$|\varphi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1. \tag{1}$$

Alice, Bob, and Charlie share a tripartite entangled state [41, 42]

$$|\phi\rangle_{234} = \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)_{234}. \tag{2}$$

The state of the whole system is

$$\begin{aligned} |\Psi\rangle_{1234} &= |\varphi\rangle_1 \otimes |\phi\rangle_{234} \\ &= \frac{1}{2}(\alpha|0\rangle_1 + \beta|1\rangle_1)(|000\rangle + |110\rangle + |101\rangle + |011\rangle)_{234}. \end{aligned} \tag{3}$$

Firstly, Alice performs a joint measurement on her two particles 1 and 2 with respect to Bell states,

$$|\Phi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{12}, \qquad |\Psi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{12}, \tag{4}$$

the particles held by Bob and Charlie collapse onto one of the following entangled states

$${}_{12}\langle\Phi^+|\Psi\rangle_{1234} = \frac{1}{2\sqrt{2}}[\alpha(|00\rangle + |11\rangle)_{34} + \beta(|10\rangle + |01\rangle)_{34}], \tag{5}$$

$${}_{12}\langle\Phi^-|\Psi\rangle_{1234} = \frac{1}{2\sqrt{2}}[\alpha(|00\rangle + |11\rangle)_{34} - \beta(|10\rangle + |01\rangle)_{34}], \tag{6}$$

$${}_{12}\langle\Psi^+|\Psi\rangle_{1234} = \frac{1}{2\sqrt{2}}[\alpha(|10\rangle + |01\rangle)_{34} + \beta(|00\rangle + |11\rangle)_{34}], \tag{7}$$

$${}_{12}\langle\Psi^-|\Psi\rangle_{1234} = \frac{1}{2\sqrt{2}}[\alpha(|10\rangle + |01\rangle)_{34} - \beta(|00\rangle + |11\rangle)_{34}]. \tag{8}$$

Table 1 Relation between the local unitary operations and the measurement results. M_{12} denotes the result of measurement on particles 1 and 2 performed by Alice, M_3 denotes measurement results on particle 3 performed by Bob, $|\phi\rangle_4$ denotes the collapsed state of qubit 4 after Bob's single-particle measurement, U_c $(c = 0, 1, 2, 3)$ denotes Charlie's unitary operation on particle 4

M_{12}	M_3	$\|\phi\rangle_4$	U_c
$\|\Phi^+\rangle_{12}$	$\|0\rangle_3$	$\alpha\|0\rangle + \beta\|1\rangle$	U_0
	$\|1\rangle_3$	$\alpha\|1\rangle + \beta\|0\rangle$	U_2
$\|\Phi^-\rangle_{12}$	$\|0\rangle_3$	$\alpha\|0\rangle - \beta\|1\rangle$	U_1
	$\|1\rangle_3$	$\alpha\|1\rangle - \beta\|0\rangle$	U_3
$\|\Psi^+\rangle_{12}$	$\|0\rangle_3$	$\alpha\|1\rangle + \beta\|0\rangle$	U_2
	$\|1\rangle_3$	$\alpha\|0\rangle + \beta\|1\rangle$	U_0
$\|\Psi^-\rangle_{12}$	$\|0\rangle_3$	$\alpha\|1\rangle - \beta\|0\rangle$	U_3
	$\|1\rangle_3$	$\alpha\|0\rangle - \beta\|1\rangle$	U_1

After the Bell-state measurement of the particles 1 and 2, the quantum information is transferred to the entangled state which is shared between Bob and Charlie due to the entanglement swapping. The distribution of quantum information is completed.

We can see that neither Bob nor Charlie can recover the state $|\phi\rangle_1$ by any general operations on their respective sides without communicating between themselves. In order to get the information, they must cooperate and only one of them can possess the final qubit for the no-cloning theorem [4]. Without loss of generality, we assume Alice obtains the state $|\Phi^+\rangle_{12}$ after her Bell-state measurement and then she publicly announces the measurement result. Now the entangled state that Bob and Charlie share can be written as following according to (5)

$$\begin{aligned}|\phi\rangle_{34} &= \frac{1}{2\sqrt{2}}[\alpha(|00\rangle + |11\rangle)_{34} + \beta(|10\rangle + |01\rangle)_{34}]\\ &= \frac{1}{4}[|0\rangle_3(\alpha|0\rangle + \beta|1\rangle)_4 + |1\rangle_3(\alpha|1\rangle + \beta|0\rangle)_4].\end{aligned} \tag{9}$$

Suppose Alice assigns Charlie to reconstruct the state. If Bob agrees to help Charlie obtain the original state, he should perform a single particle measurement on his qubit 3 and announces his measurement result. If Bob's measurement result is $|0\rangle_3$, then Charlie's qubit 4 is collapsed into the state $\alpha|0\rangle_4 + \beta|1\rangle_4$. This state is exactly the original state $|\varphi\rangle$ and Charlie needs to do nothing. While Bob's measurement result is $|1\rangle_3$, the qubit 4 is projected onto $\alpha|1\rangle_4 + \beta|0\rangle_4$, then Charlie can reconstruct the original state on his qubit 4 by performing an unitary operation $U_2 = |0\rangle\langle 1| + |1\rangle\langle 0|$, which is one of the unitary operations $\{U_i\}$ $(i = 0, 1, 2, 3)$

$$\begin{aligned}U_0 &= |0\rangle\langle 0| + |1\rangle\langle 1|, \qquad U_1 = |0\rangle\langle 0| - |1\rangle\langle 1|,\\ U_2 &= |0\rangle\langle 1| + |1\rangle\langle 0|, \qquad U_3 = |0\rangle\langle 1| - |1\rangle\langle 0|.\end{aligned} \tag{10}$$

Thus Charlie can reconstruct the state $|\varphi\rangle_4$ with the help of Bob. For the other cases, the relation between the result of the measurements done by Alice and Bob and the local unitary operations with which Charlie reconstructs the unknown quantum information $|\varphi\rangle_1$ is shown in Table 1.

Let us now discuss the security of this scheme in the following. Suppose there is an eavesdropper (say Eve) who wants to gain the quantum information transmitted by Alice by entangling an ancilla with the quantum channel during the particle distribution process. Assume all the three participants are unaware of this attack by Eve; then after Alice makes a Bell measurement, the combined state of Bob, Charlie, and Eve collapses into a three-partite entangled state. However, after Bob performs the single-particle measurement, the Charlie-Eve system collapses into a product state, leaving Eve with no information about the unknown qubit. To see this scenario more explicitly, let us assume that Eve has managed to entangle an ancilla $|0\rangle_5$ to the qubit of the entangled channel. If Alice performs a measurement in the first basis $|\Phi^+\rangle_{12}$, then the combined state of Bob, Charlie, and Eve is as following

$$\alpha(|000\rangle + |110\rangle)_{345} + \beta(|100\rangle + |010\rangle)_{345}. \tag{11}$$

If Bob performs a measurement in the basis $|0\rangle_3$, then the Charlie-Eve system collapses onto $(\alpha|0\rangle_4 + \beta|1\rangle_4)|0\rangle_5$. Hence, Eve's state is unaltered, leaving no chance for her to gain any information about the unknown qubit state, due to the fact that entanglement is monogamous [43].

In another case, we assume one of users (say, Charlie) is dishonest and he will cooperate with Eve or he is Eve himself. He can capture the channel-qubit which Alice sends to Bob and then sends Bob a qubit he has prepared before. In this way, only when Alice designates Charlie to obtain the state, he can eavesdrop the state and his cheating will go undetected. If Alice designates not Charlie but Bob to reconstruct the state, then Charlie's eavesdropping behavior can be detected. Because Charlie does not know Alice's measurement outcome and therefore the qubit that he sends to Bob is not in the correct quantum state. So the state reconstructed by Bob will differ from the state Alice has sent. When Alice and Bob compare a small part of the states publicly, the eavesdropping will be discovered.

3 QIS of an Arbitrary Two-Particle Entangled State via Two Tripartite Entangled States

Suppose that the unknown arbitrary two-particle state belongs to Alice which described as

$$|\phi\rangle_{12} = \alpha|00\rangle_{12} + \beta|01\rangle_{12} + \gamma|10\rangle_{12} + \delta|11\rangle_{12}, \tag{12}$$

where 1 and 2 are the two particles in the state $|\phi\rangle_{12}$, and $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$. Alice, Bob and Charlie share two tripartite entangled states $|\phi\rangle_{a_1b_1c_1}$ and $|\phi\rangle_{a_2b_2c_2}$ which are described as following

$$|\phi\rangle_{a_1b_1c_1} = \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)_{a_1b_1c_1}, \tag{13}$$

$$|\phi\rangle_{a_2b_2c_2} = \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)_{a_2b_2c_2}, \tag{14}$$

here Alice possesses particles a_1 and a_2, Bob possesses particles b_1 and b_2, and particles c_1 and c_2 belong to Charlie. The state of the composite quantum system composed of eight particles is

$$|\Psi\rangle = |\phi\rangle_{12} \otimes |\phi\rangle_{a_1b_1c_1} \otimes |\phi\rangle_{a_2b_2c_2}. \tag{15}$$

Now let Alice perform the Bell-state measurements on the particles 1 and a_1, particles 2 and a_2, respectively. Without loss of generality, we suppose Alice's Bell-state measurement results on particles 1 and a_1, 2 and a_2 are all in the state $|\Phi^+\rangle$, i.e., $|\Phi^+\rangle_{1a_1} = |\Phi^+\rangle_{2a_2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Then the remained four particles will collapse onto the state $|\Psi\rangle_r$

$$
\begin{aligned}
|\Psi\rangle_r &= {}_{2a_2}\langle\Phi^+|_{1a_1}\langle\Phi^+|\Psi\rangle \\
&= \frac{1}{8}[\alpha(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle) \\
&\quad + \beta(|0010\rangle + |0001\rangle + |1110\rangle + |1101\rangle) \\
&\quad + \gamma(|1000\rangle + |1011\rangle + |0100\rangle + |0111\rangle) \\
&\quad + \delta(|1010\rangle + |1001\rangle + |0110\rangle + |0101\rangle)]_{b_1c_1b_2c_2}. \qquad (16)
\end{aligned}
$$

From (16), we can see that after twice Bell-state measurement performed by Alice the quantum information has been transferred to Bob and Charlie successfully via entanglement swapping. If Bob would like to help Charlie, he should make twice single-particle measurements on particles b_1 and b_2, respectively. After above measurements, Bob sends the measurement results to Charlie via a classical channel. Since $|\Psi\rangle_r$ can be rewritten as

$$
\begin{aligned}
|\Psi\rangle_r = \frac{1}{8}[&|0\rangle_{b_1}|0\rangle_{b_2}(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle)_{c_1c_2} \\
&+ |0\rangle_{b_1}|1\rangle_{b_2}(\alpha|01\rangle + \beta|00\rangle + \gamma|11\rangle + \delta|10\rangle)_{c_1c_2} \\
&+ |1\rangle_{b_1}|0\rangle_{b_2}(\alpha|10\rangle + \beta|11\rangle + \gamma|00\rangle + \delta|01\rangle)_{c_1c_2} \\
&+ |1\rangle_{b_1}|1\rangle_{b_2}(\alpha|11\rangle + \beta|10\rangle + \gamma|01\rangle + \delta|00\rangle)_{c_1c_2}], \qquad (17)
\end{aligned}
$$

it is clear that, for instance, if Bob gets the result of $|0\rangle_{b_1}$ and $|1\rangle_{b_2}$, then particles c_1 and c_2 will collapse onto the state $|\phi\rangle_{c_1c_2} = (\alpha|01\rangle + \beta|00\rangle + \gamma|11\rangle + \delta|10\rangle)_{c_1c_2}$. According to Bob's measurement results, Charlie should perform the unitary operations U_0 on the particle c_1 and U_2 on the particle c_2, respectively, he will obtain the unknown arbitrary two-particle state $|\phi\rangle_{c_1c_2} = \alpha|00\rangle_{c_1c_2} + \beta|01\rangle_{c_1c_2} + \gamma|10\rangle_{c_1c_2} + \delta|11\rangle_{c_1c_2}$. Thus Charlie has reconstructed the original state of particles 1 and 2 on particles c_1 and c_2 with the help of Bob. For the other case, the relation of measurement results performed by Alice on the particles $(1, a_1)$ and $(2, a_2)$, single-qubit measurement results of Bob and the unitary operation performed by Charlie is shown in Table 2. There are total of 64 kinds.

Let us consider the security of this scheme against an eavesdropper Eve. Assume that Eve has been able to entangle an ancilla with the quantum channel. After Alice performs the measurement, if Bob performs a Bell-state measurement on his qubit, the Charlie-Eve system collapses into a product state, leaving the unknown qubit information with Charlie. On the other hand, similar to the case of quantum information splitting of one qubit, if Alice designates one of the users Bob (Charlie) to obtain the state, the other user Charlie (Bob) can not get the state without being detected. So our scheme is secure against eavesdropping and cheating.

4 QIS of an Arbitrary N-particle Entangled State via N Tripartite Entangled States

We could now generalize our scheme to the N-particle case ($N \geq 2$). Suppose that Alice has an arbitrary unknown N-qubit entangled secret state that she will send to Bob and Charlie. The original entangled state of an N-particle can be written as following

$$
|\psi\rangle_{1,2,\ldots,N} = \sum_{i_1,i_2,\ldots,i_N\in\{0,1\}} C_{i_1i_2\ldots i_N}|i_1, i_2, \ldots, i_N\rangle, \qquad (18)
$$

Table 2 The relation between the local unitary operations and the measurement results. MR denotes Alice's Bell-state measurement results on her particles $(1, a_1)$ and $(2, a_2)$, $|i\rangle_{b_1}|j\rangle_{b_2}(i, j = 0, 1)$ denotes Bob's single-particle measurement results, and $U_{c_1} \otimes U_{c_2}$ $(c_1, c_2 = 0, 1, 2, 3)$ denotes the unitary operations performed by Charlie

MR	$\|0\rangle_{b_1}\|0\rangle_{b_2}$	$\|0\rangle_{b_1}\|1\rangle_{b_2}$	$\|1\rangle_{b_1}\|0\rangle_{b_2}$	$\|1\rangle_{b_1}\|1\rangle_{b_2}$
$\|\Phi^+\rangle_{1a_1}\|\Phi^+\rangle_{2a_2}$	$U_0 \otimes U_0$	$U_0 \otimes U_2$	$U_2 \otimes U_0$	$U_2 \otimes U_2$
$\|\Phi^+\rangle_{1a_1}\|\Phi^-\rangle_{2a_2}$	$U_0 \otimes U_1$	$U_0 \otimes U_3$	$U_2 \otimes U_1$	$U_2 \otimes U_3$
$\|\Phi^+\rangle_{1a_1}\|\Psi^+\rangle_{2a_2}$	$U_0 \otimes U_2$	$U_0 \otimes U_0$	$U_2 \otimes U_2$	$U_2 \otimes U_0$
$\|\Phi^+\rangle_{1a_1}\|\Phi^-\rangle_{2a_2}$	$U_0 \otimes U_3$	$U_0 \otimes U_1$	$U_2 \otimes U_3$	$U_2 \otimes U_1$
$\|\Phi^-\rangle_{1a_1}\|\Phi^+\rangle_{2a_2}$	$U_1 \otimes U_0$	$U_1 \otimes U_2$	$U_2 \otimes U_1$	$U_3 \otimes U_0$
$\|\Phi^-\rangle_{1a_1}\|\Phi^-\rangle_{2a_2}$	$U_1 \otimes U_1$	$U_1 \otimes U_3$	$U_3 \otimes U_1$	$U_3 \otimes U_3$
$\|\Phi^-\rangle_{1a_1}\|\Psi^+\rangle_{2a_2}$	$U_1 \otimes U_2$	$U_1 \otimes U_0$	$U_3 \otimes U_0$	$U_2 \otimes U_1$
$\|\Phi^-\rangle_{1a_1}\|\Phi^-\rangle_{2a_2}$	$U_1 \otimes U_3$	$U_1 \otimes U_1$	$U_3 \otimes U_3$	$U_3 \otimes U_1$
$\|\Psi^+\rangle_{1a_1}\|\Phi^+\rangle_{2a_2}$	$U_2 \otimes U_0$	$U_2 \otimes U_2$	$U_0 \otimes U_0$	$U_0 \otimes U_2$
$\|\Psi^+\rangle_{1a_1}\|\Phi^-\rangle_{2a_2}$	$U_2 \otimes U_1$	$U_2 \otimes U_3$	$U_0 \otimes U_1$	$U_0 \otimes U_3$
$\|\Psi^+\rangle_{1a_1}\|\Psi^+\rangle_{2a_2}$	$U_2 \otimes U_2$	$U_2 \otimes U_0$	$U_0 \otimes U_2$	$U_0 \otimes U_0$
$\|\Psi^+\rangle_{1a_1}\|\Psi^-\rangle_{2a_2}$	$U_2 \otimes U_3$	$U_2 \otimes U_1$	$U_0 \otimes U_3$	$U_0 \otimes U_1$
$\|\Psi^-\rangle_{1a_1}\|\Phi^+\rangle_{2a_2}$	$U_2 \otimes U_1$	$U_3 \otimes U_0$	$U_1 \otimes U_0$	$U_1 \otimes U_2$
$\|\Psi^-\rangle_{1a_1}\|\Phi^-\rangle_{2a_2}$	$U_3 \otimes U_1$	$U_3 \otimes U_3$	$U_1 \otimes U_1$	$U_1 \otimes U_3$
$\|\Psi^-\rangle_{1a_1}\|\Psi^+\rangle_{2a_2}$	$U_3 \otimes U_0$	$U_2 \otimes U_1$	$U_1 \otimes U_2$	$U_1 \otimes U_0$
$\|\Psi^-\rangle_{1a_1}\|\Psi^-\rangle_{2a_2}$	$U_3 \otimes U_3$	$U_3 \otimes U_1$	$U_1 \otimes U_3$	$U_1 \otimes U_1$

where $\sum_{i_1 i_2 \cdots i_N \in \{0,1\}} |C_{i_1 i_2 \cdots i_N}|^2 = 1$. Alice, Bob and Charlie share N tripartite entangled states $|\phi\rangle_{a_j b_j c_j}$ $(j = 1, 2, \ldots, N)$

$$|\phi\rangle_{a_j b_j c_j} = \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)_{a_j b_j c_j}, \tag{19}$$

and Alice, Bob and Charlie hold particles a_j, b_j and c_j, respectively. The state of the whole system can be written as

$$|\Psi\rangle = |\psi\rangle_{1,2,\ldots,N}|\phi\rangle_{a_1 b_1 c_1}|\phi\rangle_{a_2 b_2 c_2} \cdots |\phi\rangle_{a_N b_N c_N}. \tag{20}$$

Alice makes N Bell-state measurements on her qubit pairs $\{1, a_1\}$, $\{2, a_2\}, \ldots, \{N, a_N\}$, and publicly announces her measurement results. After these measurements, the remaining particles $b_1, b_2, \ldots, b_N$; $c_1, c_2, \ldots, c_N$ are projected onto a 2N-partite entangled state which contains full information of the Alice's original quantum entangled state.

Suppose Alice assigns Charlie to reconstruct her original state, and asks Bob to perform the single-particle measurements on the particle $b_1, b_2, \ldots, b_N$, respectively, and then publicly announces his measurement results. As a consequence, Charlie's N qubits $c_1, c_2, \ldots, c_N$ will collapse into the state $|\eta\rangle_{c_1 c_2 \cdots c_N}$ which is given by

$$|\eta\rangle_{c_1 c_2 \cdots c_N} = \sum_{i_1, i_2, \ldots, i_N \in \{0,1\}} f_{i_1 i_2 \cdots i_N} |i_1, i_2, \ldots, i_N\rangle_{c_1 c_2 \cdots c_N}, \tag{21}$$

the coefficients $f_{i_1 i_2 \cdots i_N}$ was determined by Alice's and Bob's measurement results. In accord with Alice's and Bob's measurement results by their public announcement, Charlie can transform $|\eta\rangle_{c_1c_2\cdots c_N}$ to the desired state $|\psi\rangle_{1,2,\ldots,N}$ by applying the appropriate unitary operations on the each qubit $c_1, c_2, \ldots, c_N$. The security of the arbitrary multi-particle entangled state sharing scheme is the same as the single-and two- qubit case. An eavesdropper can be detected by publicly comparing a subset of the quantum state.

5 Conclusion

In conclusion, we have proposed two schemes for splitting quantum information of single- and two-qubit state by using tripartite entangled states as quantum channels, quantum information can be successfully recovered by two recipients provided that they cooperate with each other. Furthermore, the schemes can be generalized for splitting of an N-qubit quantum information. The schemes considered are secure against certain types of eavesdropping attacks.

References

1. Bennett, C.H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., Wootters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
2. Mattle, K., et al.: Phys. Rev. Lett. **76**, 4656 (1996)
3. Ekert, A.K.: Phys. Rev. Lett. **67**, 661 (1991)
4. Hillery, M., Bužek, V., Berthiaume, A.: Phys. Rev. A **59**, 1829 (1999)
5. Blakley, G.R.: In: Proceedings of the American Federation of Information Processing 1979 National Computer Conference, American Federation of Information Processing, pp. 313–317, Arlington, VA (1979)
6. Greenberger, D.M., Horne, M.A., Zeilinger, A.: In: Kafatos, M. (ed.) Bell's Theorem, Quantum Theory and The Conceptions of The Universe. Kluwer Academic, Dordrecht (1989)
7. Karlsson, A., Koashi, M., Imoto, N.: Phys. Rev. A **59**, 162 (1999)
8. Cleve, R., Gottesman, D., Lo, H.-K.: Phys. Rev. Lett. **83**, 648 (1999)
9. Gottesman, D.: Phys. Rev. A **61**, 042311 (2000)
10. Tittel, W., Zbinden, H., Gisin, N.: Phys.Rev. A **63**, 042301 (2001)
11. Karimipour, V., Bahraminasab, A., Bagherinezhad, S.: Phys. Rev. A **65**, 042320 (2002)
12. Chau, H.F.: Phys. Rev. A **66**, 060302 (2002)
13. Xiao, L., Long, G.L., Deng, F.G., Pan, J.W.: Phys. Rev. A **69**, 052307 (2004)
14. Singh, S.K., Srikanth, R.: Phys. Rev. A **71**, 012328 (2005)
15. Zhang, Z.J., Man, Z.X.: Phys. Rev. A **72**, 022303 (2005)
16. Deng, F.G., Long, G.L., Zhou, H.Y.: Phys. Lett. A **340**, 43 (2005)
17. Lance, A.M., Symal, T., Bowen, W.P., Sanders, B.C., Lam, P.K.: Phys. Rev. Lett. **92**, 177903 (2004)
18. Li, Y., Zhang, K., Peng, K.: Phys. Lett. A **324**, 420–424 (2004)
19. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Phys. Rev. A **72**, 044301 (2005)
20. Lance, A.M., Symal, T., Bowen, W.P., Sanders, B.C., Tyc, T., Ralph, T.C., Lam, P.K.: Phys. Rev. A **71**, 033814 (2005)
21. Gordon, G., Rigolin, G.: Phys. Rev. A **73**, 062316 (2006)
22. Zheng, S.B.: Phys. Rev. A **74**, 054303 (2006)
23. Deng, F.G., Li, X.H., Li, C.Y., Zhou, P., Zhou, H.Y.: Eur. Phys. J. D **39**, 459 (2006)
24. Wang, Z.Y., Yuan, H., Shi, S.H., Zhang, Z.J.: Eur. Phys. J. D **41**, 371 (2007)
25. Man, Z.X., Xia, Y.J., An, N.B.: Eur. Phys. J. D **42**, 333 (2007)
26. Xue, Z.Y., Yi, Y.M., Cao, Z.L.: Physica A **374**, 119 (2007)
27. Wang, Z.Y., Liu, Y.M., Wang, D., Zhang, Z.J.: Opt. Commun. **276**, 322 (2007)
28. Muralidharan, S., Panigrahi, P.K.: Phys. Rev. A **78**, 062333 (2008)
29. Karlsson, A., Bourennane, M.: Phys. Rev. A **58**, 4394 (1998)
30. Kempe, J.: Phys. Rev. A **60**, 910 (1999)
31. Murao, M., Jonathan, D., Plenio, M.B., Vedral, V.: Phys. Rev. A **59**, 156 (1999)
32. Zhang, C.W., Li, C.F., Wang, Z.Y., Guo, G.C.: Phys. Rev. A **62**, 042302 (2000)
33. Hao, J.C., Li, C.F., Guo, G.C.: Phys. Rev. A **63**, 054301 (2001)
34. Shi, B.S., Tomita, A.: Phys. Lett. A **296**, 161 (2002)

35. Gorvachev, V.N., Trubilko, A.I., Rodichkina, A.A., Zhiliba, A.I.: Phys. Lett. A **314**, 267 (2003)
36. Ye, L., Yu, L.B.: Phys. Lett. A **346**, 330 (2005)
37. Agrawal, P., Pati, A.: Phys. Rev. A **74**, 062320 (2006)
38. Cao, Y., Wang, A.M.: Opt. Commun. **275**, 470 (2007)
39. Wang, Y.H., Song, H.S.: Opt. Commun. **281**, 489 (2008)
40. Peng, Z.H., Jia, C.X.: Opt. Commun. **281**, 1745 (2008)
41. Bagherinezhad, S., Karimipour, V.: Phys. Rev. A **67**, 044302 (2003)
42. Walther, P., Resch, K.J., Zeilinger: Phys. Rev. Lett. **94**, 240501 (2005)
43. Coffman, V., Kundu, J., Wootters, W.K.: Phys. Rev. A **61**, 052306 (2000)